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2. A REVIEW OF CROP CANOPY REFLECTANCE MODELS

by

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PREFACE

ASTER Consulting Associates, Inc, 4245 Marietta Drive, Binghamton, New York, was awarded a contract (NAS 9-16505), involving approximately 1.5 months of effort, to review the crop canopy models.

The technical requirements of the contract were:

- (1) to objectively review the existing crop canopy models and their extensions for physical, biophysical and mathematical assumptions, rigor, and validity.
- (2) to review any other models that are of relevance to this area.
- (3) to include sensitivity of results to assumptions and extendability of these models for realistic conditions.
- (4) to suggest areas for improvement and the need for new models.

The author was to be the primary reviewer. This is the final report on the contract.

ABSTRACT

Various models for calculating crop canopy reflectance, in the visible and infrared wavelengths, from the optical and geometrical properties of a canopy and its constituents have been reviewed. The review includes a discussion of radiative transfer equation, and both analytical and numerical crop reflectance models which are manifestations of the solution of this equation. Recommendations are made for further work in modeling of canopy reflectance. These recommendations include:

- . extensive testing of various models using a common data base
- . development of a simple and still fairly realistic crop reflectance model, involving adaption of existing models, as a short term effort
- . development of a more realistic and numerical model as a long term effort
- . development of a canopy reflectance model with time implicit in it by combination of a canopy reflectance model with a crop growth model.
- . investigation of the potential of various canopy reflectance models in determining canopy parameters of importance from reflectance data, i.e. model inversion
- . adaption of other invertible reflectance models, e.g. thin layer system model to crop identification problem.
- . detailed study of other modeling areas relevant to crop canopy reflectance, including modeling of reflectance from single canopy component, modeling of reflectance in thermal and microwave regions and modeling atmosphere, to determine the strategy for future work.

ACKNOWLEDGEMENT:

The author gratefully acknowledges the generous assistance of Gautam Badhwar of NASA-Houston in directing the author to the key references during the initial stages of this work and for commenting on a draft of this report. Jim Smith very generously made available to the author many unpublished reports and hard to access published reports from his extensive personal library. These reports included his bibliographic work with Ranson and work on adding method with Cooper and Pitts. The author benefited considerably by his writings, both reporting original work and reviewing existing work. Finally, the author wishes to thank Forrest Hall and Bob McDonald of NASA-Houston for giving the opportunity to participate in the exciting Supporting Research Program.

I. INTRODUCTION

When daylight is incident on a vegetation (crop) canopy, it is scattered and reflected, and its direction and spectral composition are altered in a complex manner by the vegetation. Part of this altered radiation is remotely sensed by Landstat. It is hoped that with multispectral measurements one can determine crop canopy parameters which can assist in

- . crop identification
 - . crop growth stage determination
 - . crop quality or abnormality determination
- and
- . eventually crop yield calculations.

The role of crop canopy reflectance modeling towards achieving this objective is stated rather well by Suits (1981), who proposed a crop canopy model which has been developed and investigated by many scientists and is widely used. According to him:

' A canopy reflectance model provides the logical connection between the botanical features of the canopy, the geometry of the radiometric interaction and the resulting alteration in the reflected radiation. Such a model allows one to understand the reasons for the alterations and to calculate the magnitude and trends of these alterations caused by the botanical features and the geometry of the interaction. The validity of inferences as to the existence of important agronomic features from the detected altered radiation may be tested on theoretical grounds'.

Over the last 15 years or so, several canopy reflectance models have been developed. These models represent either an approximate or a numerical attempt to solve what is known as radiative transfer equation which is a macroscopic manifestation of the interaction of radiant energy with matter. In Section II, we provide a general perspective of the interaction of electromagnetic waves with crop canopies including radiative transfer theory. In Section III, we summarize those crop reflectance models which are based on approximate but analytical solution of the radiative transfer equation. We shall refer to them as analytical models. In Section IV, we summarize numerical crop reflectance models where an attempt is made to numerically solve the radiative transfer equation. In the discussion of both types of models, we will emphasize the key assumptions made, and point out agronomic variables which are used in the calculation of reflectance, with the hope that it will assist the reader in assessing the usefulness of a model in estimating agronomic variables from the reflectance data. For details of various models, the readers are referred to the original papers. The readers may also find an unpublished report 'MRS Literature Survey of Bidirectional Reflectance and Atmospheric Corrections II. Bidirectional Reflectance Studies Literature Review' prepared for NASA-Goddard Space Flight Center, Greenbelt, MD. by J.A. Smith and K.J. Ranson useful. This extensive report (about 200 pages) dated August 1979 provides an excellent comprehensive review of previous work in scene bidirectional reflectance, an extensive bibliography and abstracts of key papers.

In Section V, we will recommend a strategy for further work in the areas of crop reflectance modeling. This strategy includes comparative testing of various models, initiation of work involving inverting the models, i.e. to develop procedures for obtaining agronomic variables from the reflectance data using these models, and the desirability of new models.

II. INTERACTION OF ELECTROMAGNETIC WAVES WITH CROP CANOPIES - GENERAL PERSPECTIVES

The theoretical basis for understanding the interaction between electromagnetic radiations and crop canopies is the radiative transfer theory, also called transport theory. It is a macroscopic theory of the interaction of radiant energy with matter. It describes the observed phenomena of light scattering, absorption, and polarization effects, but without regard to the classical electromagnetic theory. This theory has been developed, and applied extensively by astrophysicists, earth and atmospheric scientists for studying stellar or planetary atmosphere, earth surfaces and oceans. One of the classic and encyclopedic work in the field is due to a well known astrophysicist, S. Chandrasekhar (1950). Since then many texts and monographs have appeared emphasizing different aspects of the theory, including a more recent and rather comprehensive set of two volumes by Ishimaru (1978 a, b).

The mathematical apparatus of the radiative transfer theory, though conceptually straightforward, can not be easily applied to the vegetation case because of a number of unusual features of the vegetation. To appreciate these, from the pedagogical point of view, it is desirable to provide a quick review of the radiative transfer theory, adapted from Ishimaru (1978a) and to a lesser extent from Smith and Ranson (1979).

The transport theory describes the propagation of intensities in randomly distributed particles in terms of specific intensity $I(\vec{r}, \hat{s})$, which is in general function of position \vec{r} and direction \hat{s} in a three-dimensional space. Specific intensity, also called radiance or brightness, is the average power flux density within a unit frequency band centered at frequency ν within a unit solid angle and is measured in $\text{W m}^{-2} \text{Sr}^{-1} \text{H}^{-1}$ (Watts/meter²/steradian or solid angle/Hertz).

This quantity satisfies the so called equation of transfer

$$\frac{dI(\vec{r}, \hat{s})}{ds} = -\rho \sigma I(\vec{r}, \hat{s}) + \rho \sigma J(\vec{r}, \hat{s}) + \epsilon(\vec{r}, \hat{s}) \quad (2.1)$$

In this equation, ρ is the number of particles per unit volume with which the incident radiation interacts and σ is the total of scattering and absorption cross sections of particles (i.e. each particle absorbs/scatters the power σI). Thus, the first term in the right hand side of Eq. (2.1) reflects the decrease in I due to absorption and scattering by particles in volume ds . The second term represents the portion of the specific intensity incident on this volume from all other directions due to scattering from particles outside this volume. The third term represents the increase in I due to emission from within the volume ds .

To calculate J one defines a so called phase function^{*} $p(\hat{s}, \hat{s}')$ which is the probability that radiance at \hat{s}' in a direction \hat{s} , will be scattered into a solid angle about \hat{s} . It is defined by

$$p(\hat{s}, \hat{s}') = \frac{4\pi}{\sigma} |f(\hat{s}, \hat{s}')|^2 \quad (2.2)$$

$$\frac{1}{4\pi} \int |f(\hat{s}, \hat{s}')|^2 d\omega = \frac{\sigma_s}{\sigma} \quad (2.3)$$

where f is the scattering amplitude. J is then given by

$$J(\vec{r}, \hat{s}) = \frac{1}{4\pi} \int p(\hat{s}, \hat{s}') I(\vec{r}, \hat{s}') d\omega' \quad (2.4)$$

^{*}The name "phase function" has its origin in astronomy where it refers to lunar phases. It has no relation to the phase of a wave

where the integration over all w' is taken to include the contributions from all directions \hat{s}' .

In the above equations, the particle density and size can be at different locations and therefore ρ , σ and p can be functions of \vec{r} . It is sometimes convenient to measure the distance in terms of non-dimensional "optical" distance τ defined by

$$\tau \equiv \int \rho ds$$

With this definition, from Eqs. (2.1) and (2.4)

$$\begin{aligned} \frac{dI(\tau, \hat{s})}{d\tau} = & -I(\tau, \hat{s}) + \frac{1}{4\pi} \int p(\hat{s}, \hat{s}') I(\tau, \hat{s}') d\omega' \\ & + \frac{c(\vec{r}, \hat{s})}{\sigma \rho} \end{aligned} \quad (2.5)$$

This is the basic integro-differential equation which needs to be solved for I for a vegetation canopy. The solution involves two major steps:

(1) Calculation or Specification of the phase function $p(\hat{s}, \hat{s}')$ in terms of vegetation canopy properties. For any applications, this is a rather difficult task. It is even more difficult for the canopy because vegetation is extremely heterogeneous and complex and the canopy can not be treated either as a regular or completely random medium. Also, the scattering and absorbing elements of vegetation canopy, namely, leaves, stalks, flowers, etc. are very large compared to the molecules and the aerosol components of air, and are characterized by a relatively high absorption coefficient; about 0.85 for the photosynthetically active radiation (PAR) and about 0.15 for the near IR radiation.

(2) Solution of the Equation for a given phase function and boundary condition. One of the procedures for solving an integro-differential equation like (2.5) is to substitute an initial guess for I in the right hand side and then integrate the equation, subject to the boundary conditions on I , to get a new I which is then used again in the right hand side to get a new solution. This iterative procedure is continued till the value of I does not change (within a desired accuracy). For vegetation (unlike the top of the atmosphere), the upper surface of the plant cover is exposed both to the direct specular radiation and the diffuse flux of the scattered radiation from the sky, leading to a somewhat difficult boundary condition. Also, the optical thickness of the plant canopy is substantially higher (8-10) than that for the atmosphere (0.2-0.6) leading to a slow convergence of the iterative procedure.

One simplification which vegetation canopy provides is that the emission from within the canopy is negligible* and hence $\epsilon(\vec{r}, \vec{\Omega}) = 0$ in Eq. (2.5).

The unique features of the vegetation canopy have been recognized by the various investigators. Generally, simpler problems are solved by imposing abstractions on the shape or boundary of the medium and on the form of the phase function. We will attempt to delineate them in the next section. Before we do so, we will like to note the following two points:

(a) Canopy as a parallel plane medium:

One of the simplifications made by the majority of crop canopy reflectance models is to approximate medium (canopy) with a parallel-plane infinitely extended medium, i.e. the one in which the medium can be split up into distinct layers (one or more) in which the optical and structural properties are constant. In this case, the specific intensity is a function of one dimension z perpendicular

* See however Ellenson and Amundson (1981) who found delayed light emission from soybean leaves exposed to sulfur dioxide and used it as a means to detect plant stress.

to the layer and angles θ and ϕ defining the direction of incident beam.

For this case, the radiative transfer equation may be re-expressed as

$$\mu \frac{dI(\tau, \mu, \phi)}{d\tau} = I(\tau, \mu, \phi) + K(\tau, \mu, \phi) \quad (2.6)$$

$$\text{where } K \equiv \frac{1}{4\pi} \int_{-1}^1 d\mu' \int_0^{2\pi} d\phi' p(\mu, \phi; \mu', \phi') I(\tau, \mu', \phi') \quad (2.7)$$

$$\mu = \cos \theta, \quad dz = \mu ds, \quad d\tau = \rho \sigma dz$$

and τ is the optical distance in the z direction ($dz = \mu ds$)

Eq.(2.7) can be formally integrated to give

$$I(\tau, \pm\mu, \phi) = I(\tau_0, \pm\mu, \phi) e^{-(\tau-\tau_0)/\mu} + \frac{1}{\mu} \int_{\tau_0}^{\tau} K(\tau', \pm\mu, \phi) e^{-(\tau-\tau')/\mu} d\tau' \quad (2.8)$$

Basically, this equation states that the upward (downward) radiance at optical path τ , is a result of the upward (downward) attenuated radiance at τ_0 plus that scattered into the beam along the path between τ and τ_0 .

Even for the simple geometry, no closed form solution has been found for a general arbitrary phase function and one has to resort to computer based numerical solutions. Various canopy reflectance models either make some further approximations or find numerical solutions.

(b) Polarization Effects:

The above formulation of the radiative transfer theory excludes the polarization effects. Such effects may be important for some classes of materials, e.g., vegetation canopies with waxy leaves - pine needles, rhododendron,

holly - which often produce a strong specular reflection or glaze.

To include the polarization effect one replaces $I(\vec{r}, \hat{s})$ by a vector $I(\vec{r}, \hat{s}, \hat{t})$ whose components are the so called Stokes parameters I_1, I_2, U and V (Ishimaru, 1978a) defined by

$$I_1 = |E_1|^2, \quad I_2 = |E_2|^2 \quad (2.9a)$$

$$U = 2\text{Re} (E_1 E_2^*) = 2 a_1 a_2 \cos \delta \quad (2.9b)$$

$$V = 2\text{Im} (E_1 E_2^*) = 2 a_1 a_2 \sin \delta, \quad \delta \equiv \delta_2 - \delta_1 \quad (2.9c)$$

where

$$E_1 = a_1 \exp (-i\delta_1 + i k z) ; \quad E_2 = a_2 \exp (-i\delta_2 + i k z) \quad (2.9d)$$

are the phasor representations of the electric field components E_x and E_y in the mutually perpendicular directions ($E_x = a_1 \cos (wt - kz + \delta_1)$, $E_y = a_2 \cos (wt - kz + \delta_2)$) of a plane wave propagating in the z direction.

Whether one should stress polarization effects or not in the modeling effort is not clear at this time. Egan (1968, 1970) suggests that discrimination potentials may exist in the asymmetric depolarization effects as a function of view angle. Curran (1981) argues that the polarized visible light is an indication of scene roughness which is a function of vegetation amount. He also finds a linear relationship between polarized visible light and vegetation amount.

Vanderbilt (1980) had modeled plant canopy polarization response and Vanderbilt, Biehl, Robinson, Bauer and Vanderbilt (1980) have measured the linear polarization of light by wheat canopies. These scant but definitive works do suggest investigating polarization effects, especially in the context

of ground based reflectance measurements.

We now move on to the discussion of the various crop reflectance models.

III. ANALYTICAL CROP REFLECTANCE MODELS

We will now provide an overview of various crop reflectance models, analytical ones in this section and the numerical ones in the next section, with emphasis on the key assumptions and the agronomic variables used in the models. In Section V, we will compare and contrast various models and recommend a strategy for future work.

(1) Models Directly Based on Kubelka-Munk (KM) Theory

As an alternative to the numerical solution of the transport equation, in 1931, Kubelka and Munk proposed an approximate theory for a parallel plane random medium (See (a) of Section II). The light flux is described in terms of two nonochromatic fluxes E_- and E_+ , travelling in the downward and upward directions, perpendicular to the plane of the medium. The variations of these fluxes within the medium are described by means of two parameters α and γ which are absorption and scattering coefficients, respectively, assumed same for E_- and E_+ . More specifically,

$$\frac{dE_-}{d(-\tau)} = -(\alpha + \gamma) E_- + \gamma E_+ \quad (2.1.1a)$$

$$\frac{dE_+}{d\tau} = -(\alpha + \gamma) E_+ + \gamma E_- \quad (2.1.1b)$$

The first term in the right hand side of Eq. (2.1.1a) states that the downward flux decreases due to its absorption and scattering, while it increases due to scattering of upward flux. Eq. (2.1.1b) has similar physical interpretations.

The relationships between fluxes* E_- and E_+ and specific intensity I introduced in the preceding section are

$$E_+ = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I(\tau, \mu, \phi) \mu \sin \theta d\theta d\phi \quad (2.1.2a)$$

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$$E_- = \int_{\phi=0}^{2\pi} \int_{\theta=\pi/2}^{\pi} I(r, -\mu, \phi) \mu \sin \theta d\theta d\phi \quad (2.1.2b)$$

The attractive feature of the KM theory is that it requires no more than simple algebraic operations to calculate the flux at any point in the medium. This attraction is in a large part responsible for the application of the KM theory in many applications of the radiative transfer theory. The drawbacks of the theory are the requirements of empirical determination of coefficients α and γ , and lack of theoretical basis and precise understanding of range of its validity. (See also Kortüm, 1969).

Subsequent to the initial proposal, a number of authors refined the KM theory and compared it with experimental data. The general conclusion of these investigations is that this two flux (E_- and E_+) theory adequately describes the experimental data if

- . illumination is diffused (isotropic light) and
- . medium is dull so that the light is diffusely scattered (ideal diffusing medium)

It is not applicable if

- . a collimated or specular beam is incident upon the medium
- . medium consists of several distinct components that result in anisotropic reflectance.

In connection with vegetation canopy, it should be noted that for a cloudy day, the canopy is illuminated by sun light which is predominantly diffused. On a clear day, the sun light is predominantly collimated. However, it becomes diffused very quickly as it enters the canopy especially when the canopy is reasonably dense. Further, the canopy does have distinct components which lead

* E_- and E_+ at the top of the plane parallel medium are also known as irradiance and radiant emittance (or radiant excitance), respectively.

to a isotropic reflectance e.g., the reflectance difference between upper and lower surfaces of many plant leaves, bidirectional scattering effects of individual leaves, inhomogenous distribution of leaf orientation, etc.

The case of collimated beam can be included by adding two more fluxes, upward (F_+) and downward (F_-) collimated fluxes, resulting into the so called KM four flux theory (Ishimaru, 1978a). The equations which describe this extension are

$$\frac{dE_-}{d(-\tau)} = -(\alpha + \gamma) E_- + \gamma E_+ + S_1 F_- + S_2 F_+ \quad (2.1.3a)$$

$$\frac{dE_+}{d(\tau)} = -(\alpha + \gamma) E_+ + \gamma E_- + S_1 F_+ + S_2 F_- \quad (2.1.3b)$$

$$\frac{dF_-}{d(-\tau)} = -(k + S_1 + S_2) F_- \quad (2.1.3c)$$

$$\frac{dF_+}{d\tau} = -(k + S_1 + S_2) F_+ \quad (2.1.3d)$$

where k is the absorption coefficient for collimated beams, and S_1 and S_2 are the coefficients of scattering from a collimated beam into a diffuse flux in the same direction and in the opposite direction, respectively.

If one assumes that the collimated flux is only in the downward direction ($F_+ = 0$) the four flux KM theory is a three flux theory and is called as the Duntley theory (Duntley, 1942) involving 5 parameters.

The inclusion of anisotropic canopy reflectance in the theory is rather

difficult. One aspect, namely, asymmetry between optical properties of the two sides of the canopy components, can be included by using different absorption and scattering coefficients for fluxes in the upward and downward directions. For example, Park and Deering (1982) (see also Bunnik and Verhoef (1974)), for the case of diffuse flux only, modify the KM equations to the following equations:

$$\frac{dE_-}{d(-\tau)} = (\alpha_- + \gamma_-) E_- + \gamma_+ E_+ \quad (2.1.4a)$$

$$\frac{dE_+}{d(-\tau)} = -(\alpha_+ + \gamma_+) E_+ + \gamma_- E_- \quad (2.1.4b)$$

where α_- and α_+ are absorption coefficients associated with E_- and E_+ respectively and γ_- and γ_+ are the respective scattering coefficients.

Before we discuss the applications of the KM theory to vegetative canopy reflectance, we should note the following:

(a) The hemi-spherical reflectance of the canopy can be calculated by forming the ratio of the upward directed diffuse flux to the downward directed flux (specular + diffused) at the top of the canopy. Here it should be noted that since the diffuse flow within the canopy is assumed to be isotropic, the canopy reflectance is presumed to be Lambertian.

* A Lambertian surface is one for which the specific intensity $I(\vec{r}, \hat{s})$ is independent of the direction \hat{s} , i.e. radiation is isotropic. For this case from Eq. (2.1.2a)

$$E_+ = I(\tau) \int_{\theta=0}^{\theta=\pi/2} 2\pi \left(\frac{\cos^2 \theta}{2} \right) d\theta = \pi I(\tau) \quad (2.1.5)$$

i.e. the ratio of irradiance to specific intensity (also known as spectral

radiance) is π .

(b) Variable τ can represent a canopy variable other than optical thickness (provided this new variable either is or assumed to be proportional to optical thickness) with accompanying slightly different interpretation of parameters. For example, Park and Deering (1982) set τ as biomass per unit area. Allen, Gayle, and Richardson (1970) set τ equal to cumulative leaf area index (LAI) defined as cumulative one-sided leaf area per unit ground area from the top surface of a stand to a plane at a given distance above the ground.

Coming back to the specific use of the KM theory in connection with vegetative canopy reflectance, it seems that investigators at the U.S. Department of Agriculture at Weslaco, Texas were the first ones to make such use of the theory. Allen and Brown (1965) applied it to radiation in corn canopy and noted its inadequacy to account for the specular component of radiation and for the observed variation of plant canopy reflectance with the sun angle. A good test of the applicability of the KM theory was carried out by Allen and Richardson (1968). They measured the reflectance (R) and transmittance (T) of stacked mature cotton leaves (normal and dehydrated) over the spectral range 0.5 - 2.5 μ and found that the deviation between theory and experiment is only about 1% , assuming certain wavelength (λ) dependence of α and γ (in their notation K and S) for cotton leaves. They also gave an explicit solution to the KM equations for an actual plant canopy.

In order to account for illumination of canopy by direct sunlight and phenomena produced by sun angle, leaf orientation, or other attributes, 2 parameters KM theory was replaced by 5 parameters Duntley theory (Allen, Gayle and Richardson, 1970). Recalling that this theory assumes no upward specular light, its application assumes that the specular light incident on the leaves as well as on the soil background is reflected as diffuse light. The effect of the sun zenith angle, θ , is included by assuming a $\sec \theta$ dependence of

specular light attenuation coefficient, $k + S_1 + S_2$ (see Eq. 2.1.3c). The Duntley equations fit the near infra-red experimental data on corn canopy very well.

In Table 2.1.1 are given the values of various fitted parameters that specify near-IR irradiance in a corn canopy.

Table 2.1.1

Fitted parameters that specify near-IR irradiance in a corn canopy (Allen, Gayle, and Richardson, 1970).

Parameters	Fit 1	Fit 2
α	0.000	0.035*
γ	1.369	0.736*
k	0.000	0.125
S_1	0.609	0.281
S_2	0.978	0.297
Standard deviation	3.2%	3.7%

* Laboratory values for single leaves at $\lambda = 1\mu$.

Fit 2 corresponds to the case when the laboratory measurements of optical constants, α and γ , for a single corn leaf are used as constraints. Fit 1 corresponds to no constraint case and suggests that the best fit to the near infra-red transmittance occurs when zero absorptance is assumed for the canopy ($\alpha = 0$, $k = 0$).

More recently, the KM two flux theory, with unequal absorption as well as scattering coefficients for upward and downward flux, has been applied by Park and Deering (1982) to the observed reflectance at 0.68μ as a function of dry biomass for alfalfa and shortgrass prairie canopies (for alfalfa different

biomass levels were created through selective thinning within a larger uniform stand of alfalfa and the observations were made under various sky and illumination conditions). They tried to fit the data to several models reflecting various relationships between four parameters α_- , γ_- , α_+ and γ_+ (eq. (2.1.3)). For alfalfa, no noticeable differences were found in the fits for the following two models

Model A: $r = \gamma_-/\gamma_+ \gg 1$ (Downward flux scattering much more than that of upward flux)

Model B: $\alpha_- = \alpha_+$, $\gamma_- = \gamma_+$ (i.e. KM 2 flux theory)

For shortgrass prairie no definitive best model could be established presumably because of the large scatter of the data.

They also found that

- optical parameters varied for differing illumination conditions and for sure depended on the sun angle.

We should add that they found the general model (with four parameters α_- , α_+ , γ_- , γ_+) "inefficient for computation" and hence did not use it in their analysis. It should be emphasized that the best fit parameters for alfalfa and shortgrass prairie canopies were quite distinct from each other (See Table 2.2.2).

Table 2.2.2

Parameter	Alfalfa	Shortgrass prairie
$K(10^6 \text{ hectare/kg})$	444-741	18.6
R_v	.019-.033	≈ 0
soil reflectance, R_b	.165-.258	$\approx .186$
$D \equiv (\alpha_- - \alpha_+ + \gamma_- - \gamma_+)/2$		
$K \equiv (D^2 + \alpha_- \alpha_+ + \alpha_- \gamma_+ + \alpha_+ \gamma_-)1/2$		
$R_v \equiv \{(\alpha_- + \alpha_+ + \gamma_- + \gamma_+) / (2\gamma_+)\} - (K/\gamma_+)$		

and hence

. The KM theory based models, in principle, could be used for automatic crop identification and possibly also assessment.

We should point out that though the KM model has been explicitly studied when canopy has only one layer, i.e. it can be assumed to be a homogenous layer, it can easily be extended for the case of many layers, with each layer being characterized by its own set of parameters. This extension requires ensuring the continuity of upward and downward fluxes at the boundaries of various layers as has been done by Chance and LeMaster (1977) in connection with the study of the Suits Model which we will describe next.

2. Suits Model and Its Refinements

In the KM theory, Duntley theory or in spirit equivalent model due to Allen, Gayle and Richardson, since the upward diffuse flux E_+ is isotropic or Lambertian by definition, the calculated reflectance is independent of the view angle while it does depend on the solar incidence angle. Also, canopy geometry is not taken into consideration.

The first canopy reflectance model that is truly bidirectional, i.e. dependent on both the solar zenith angle and on the zenith angle and on the zenith angle of observation was developed by Suits (1972 a,b,c; Suits and Safir, 1972). This was achieved by solving radiative transfer equation more exactly yielding more realistic non-Lambertian canopy radiance. The model also expressed parameters of the KM theory in terms of parameters, defining the canopy geometry and optical properties of the canopy components, which can be measured in the field. Canopy layers containing mixed components (e.g., leaves, flowers, and stalks) can also be included. In Suits' model, the canopy is idealized as a homogeneous mixture of horizontal and vertical diffusely reflecting and transmitting flat panels that are considered to replace the original leaves by taking their horizontal and (two orthogonal) vertical projections. For this simple geometry, the KM model parameters are expressed in terms of the following parameters*

σ_h = average area of the projection of a leaf (or another vegetative component) on a horizontal plane.

σ_v = average area of the leaf when projected onto two orthogonal vertical planes. (see Fig.3.1)

n_h = number of horizontal projections per unit volume.

n_v = number of vertical projections per unit volume.

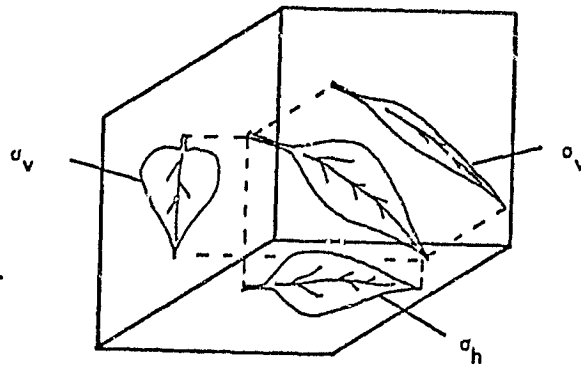
ρ = hemi-spherical reflectance of a leaf.

τ = hemi-spherical transmittance of a leaf.

θ = polar angle for incident specular flux.

*Suits does not give the explicit derivation of these expressions in his publications. See Slater (1980) for these derivations.

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The horizontal projection σ_h and the two vertical projections σ_v of a leaf.

Figure 3.1

ρ and τ are assumed to be the same for both sides of a leaf.

If more than one kind of vegetative components exist in a canopy layer, then the values of the KM model parameters are obtained for each type separately and then added together to obtain the value for the layer.

The radiative transfer equation is approximately solved by making an initial guess for the total radiance field I . For this purpose I is assumed as a sum of upward and downward diffused field and an attenuated downward specular fields.

$$I = I_+ + I_- + I_s$$

This initial guess is obtained by solving the resulting simplified radiative transfer equations which are the KM 3 flux or Duntley equations. The phase function is also assumed to be as a sum of three parts (u, v, w' ; Suits 1981^{*}) corresponding to these three fields. Each of these phase function is explicitly expressed in terms of the canopy geometric parameters and the optical properties of the canopy given earlier in this subsection and the polar view direction θ_v and azimuthal angle between sun and view direction ψ' . The initial guess together with the phase function is then used to calculate the source function K in Eq. (2.7). This value of K is substituted in Eq. (2.6) which is then solved to get an updated estimate of I along a particular direction. In principle, this iterative procedure could be used to generate a solution to any desired degree of accuracy. However, Suits stops at the first iteration.

The procedure leads to a closed form expression for canopy reflectance leading to easily implemented computer codes. (There is now available a code in BASIC language which can be run on an Apple II microcomputer). Suits (1972a; see also Slater, 1980), and Chance and Cantu (1975) give explicit expressions for the canopy reflectance for single layer and two layer canopies. The only additional parameters these expressions have are the soil reflectance ρ_s , thickness of

* The relation for w' had a missing factor in the original publication (Suits, 1972a).

various layers in the canopies and relative fractions of diffuse and specular incident lights. Also, the parameters σ_h , σ_v , n_h , n_v occur only in the combination $\sigma_h n_h$ and $\sigma_v n_v$.

The Suits model has been tested against experimental data by Suits and his colleagues at his institution (Suits, 1972a, b, c; Suits and Safir, 1972; Colwell, 1974) and by many other investigators at other institutions, most notably at Pan American University (Chance and Cantu, 1975; Chance, 1977; Chance and LeMaster, 1977; 1978; LeMaster and Chance, 1978; LeMaster, Chance and Wiegand, 1980), and at Netherlands Interdepartmental Working Community for the Application of Remote Sensing Techniques, NIWARS (Bunnik & Verhoef, 1974; Verhoef & Bunnik, 1975, 1976, 1981; Bunnik, 1978). These studies provided a very good understanding of alterations in the reflectance caused by the botanical features and the geometry of the interaction (Cause-effect relationship). They also pointed out the shortcomings of the Suits model and refined and extended the model.

It is beyond the scope of this report to go into the details of the various insights in the cause-effect relationships and on the shortcomings. The readers are referred to the works cited in the preceding paragraph, especially works by Verhoef and Bunnik (1975) and by the Pan American University group. Here, we only point out that there is, in general, a good agreement between the model and the experimental data for spectral reflectance, and the effects of variations in leaf area index, average leaf inclinations and soil moisture on the reflectance could be simulated very well. For $\lambda = 600-690$ nm, with small penetration, a single layers models fits well, while for $\lambda \geq 690$ nm, with deeper penetration, a multilayer model is required. For $\lambda = 500-600$ nm, with moderate penetration, a two layer model is the best choice. (Chance and LeMaster, 1977). Here, we should point out that Chance and Cantu (1975)

developed the mathematics which enabled the extension of the Suits model for an arbitrary number of layers. Also, Bunnik and Verhoef (1974) introduced unequal optical properties at the upper and lower sides of the canopy material.

The poorest agreement between the model and the experimental data is found for very early and very late in the growing season when the green plant biomass is low and the ground cover is incomplete (LeMaster, Chance and Wiegand, 1980). At these times, the assumption of continuous and uniform canopy of the Suits model are not well satisfied. There is another situation where the plant canopy is not uniform and homogenous horizontally and the Suits model does not agree with the observation. This situation leads to the so called "row effect" and is described below (Suits, 1981).

Many crops are planted in rows by machinery. Upon emergence of the plants, the bare soil between rows is still the dominant feature which reflects incident daylight. As growth continues, the vegetation grows both higher and spread out over the inter-row regions covering the bare soil. For a considerable time during the early part of the growing season, the strips of bare soil between rows and the increasing density of vegetation along the rows become equally important in their contributions to canopy reflectance. During this time, the direction of sunlight relative to row direction will change the relative influence of vegetation and bare soil. When the sun is directed along the row direction, the bare soil is fully illuminated, but, when the sun is directed across the rows, the soil is largely in the shadow of the standing vegetation along the rows. Thus, Landstat can receive different alterations due only to the way the rows trend relative to sunlight. An inference that such layered radiation is due to a change in some important agronomic feature, could thus, be in error.

Field measurements of wheat (Jackson et al, 1979a) and soybean (Vanderbilt et al, 1981, Randon et al, 1982) show that row direction relative to the sun light

does change the character of the scattered and reflected radiation. In fact, the reflectance may change by a factor of 2 or 3 for the same canopy depending upon the direction of rows relative to sunlight.

Verhoef and Bunnik (1976) were the first ones to extend the Suits model to include the effects of rows by assuming rectangular crosssections of crops planted in rows, with random arrangement of leaves in each row and only soil between rows. According to Smith and Ranson (1979), they undertook a detailed geometrical analysis of the canopy phase function relative to direct solar flux, with shading allowed. However, the approximation is made that the diffuse flux can be treated as in the homogenous Suits model. For both types of fluxes, an appropriate view probability function is developed that is consistent with the row structure. The soil contribution to the canopy reflectance is carefully developed considering the row structure, but even then the contribution is discontinuous. Leaf angle distributions are handled via vertical and horizontal projections as in the basic Suits model. The row model was verified against spectral measurement on wheat and it did predict the angular dependence of reflectance relative to viewing azimuth.

More recently Suits (1981) included the row effects in his model for more general cross-sections of crops by essentially multiplying the KM parameters in his model by a row modulation factor depicting the geometry of the rows. He applied the model to wheat and found that the new model does give the result similar to those of field measurement and to those obtained by Verhoef & Bunnik (1976).

LeMaster and Chance (1978) found another disagreement between the model and experimental data. For Penjamo wheat, the observed reflectance at 550 and 850 nm increased with the sun zenith angle, while the Suits model predicts a decrease. Similarly, for a fixed sun angle, the reflectance increased with increase in the

observer zenith angle, contrary to decrease predicted by the Suits' model. This disagreement may be due to the assumption of Lambertian reflecting leaves used in the Suits' model for specular irradiance (LeMaster and Chance, 1978) and/or the assumption of no oblique leaves (Verhoef and Bunnik, 1981).

Suits model has been extended by the Dutch group (Verhoef and Bunnik, 1981), to include scattering and extinction functions for canopy layers containing fractions of oblique leaves according to a given leaf inclination distribution function (LIDF). In this model, called SAIL (Scattering by Arbitrarily Inclined Leaves), the Suits model simplification of canopy geometry to exclusively horizontal and vertical panels is replaced by discretization of canopy LIDF to a set of frequencies at distinct leaf inclinations θ_l . SAIL model seems to have removed the limitations of the Suits model and can predict crop reflectance for most of the leaf inclination distributions found in nature.

Another extension of the Suits model was carried out by Sadowski and Malila (1977) to forest canopies. They introduced slope angle and additional parameters characterizing ground surface. Their model is empirically calibrated and is applicable for a uniform canopy; it is not driven by tree size and spacing. The model lacks an explicit geometric component to account for shadowing. The model depicts fairly accurately the observed reflectance including the phenomenon that open canopy stands under low angles of solar illumination will have reflectance similar to a closed canopy stands under higher illumination angles (Strahler, 1981).

We conclude the discussion of the Suits model by pointing out that Chance and LeMaster (1978) proposed a light absorption model (LAM) for vegetative plant canopies based on Suits canopy reflectance model. Both of these models have the same set of experimental parameters. Reflectance model has its value in

in terms of characterizing the canopy, while once this has been done, the absorption model can be used to determine the absorption of light in the photosynthetically active region (PAR) of the spectrum (4000-7000A⁰) which in turn is required for determining the crop growth. LAM's predictions are found to be in agreement with the experimental data for Penjamo wheat in the soft dough stage (98 days from the emergence with an LAI of 3.5). Also, the variation of percent canopy absorption as a function of LAI seems to agree with the measurements on Plainsman V wheat.

IV. NUMERICAL CROP REFLECTANCE MODELS

In this section, we will briefly discuss those models in which numerical means are used to either solve completely and directly the radiative transfer equation (2.5) or (2.8) or part of it or its equivalent manifestation. The hope is that one will then be able to analyze more realistically the canopy reflection for a larger variety of canopy geometries. At the outset, we should point out that these models and their intricacies are hard to grasp even by those who have developed competitive approaches. This together with the difficulty in obtaining the computer codes which implement various models are in a large part responsible for almost an order of magnitude less use of these models as compared to the models described in the preceding section.

(1) Smith and Oliver Model

It is one of the more known numerical models proposed essentially at the same time as Suits model (Smith and Oliver, 1972; 1974; See also Slater, 1980; and Smith and Ranson, 1979). Here, a direct attack on the numerical solution of the radiative transfer equation (2.8) for a plane parallel medium is made.

In this model, the flux within the canopy is allowed to propagate in discretized (θ' , ϕ') directions rather than only in upward and downward directions as for the KM theory based models and Suits model.

The main feature of the model which allows for this generality is the calculation of the layer phase functions from the angular distribution of the foliage elements and the reflectance and transmission properties of these elements with respect to the discretized θ' , ϕ' source directions. A foliage element inclined at an arbitrary orientation with respect to the source direction permits, according to the Lambertian response, the scattering by transmission and reflection of the incident flux to upper and lower hemispherical sectors.

For each foliage inclination represented in the canopy a set of integration limits on the scattered radiation from a scatterer is defined. For a given layer the distribution of flux is then weighted by the frequency distribution of foliage inclinations occurring within the layer.

The model initiates an iterative solution of Eq. (2.8) by using the zero order flux above the canopy to generate via the phase function of the first layer the estimated flux in layer one. This is then used together with the phase function for layer two to calculate the estimated flux in layer two and so forth. Subsequently, reflection from the soil boundary generates upward moving flux, again in a set of discretized θ' , ϕ' directions. Processing is continued until all flux levels within layers reach equilibrium values.

The reported version of the model is a Monte Carlo or stochastic implementation of the above processes. The following description adapted from Smith and Oliver (1972, 1974) and Blater (1980) gives more insight into the model.

The direct solar radiation is treated as a set of independent source vectors to the canopy. For simplicity, all the vectors are divided into source sectors formed by partitioning the hemi-sphere into 10 degree inclination bands and further subdividing the bands to form 20 degrees azimuthal sectors. The midpoint of the sector is taken as the direction of the diffuse flux from that sector. The interaction with the canopy of each of these initial 181 radiation sources is treated independently.

The diffuse flux resulting from the interaction of global radiation with a canopy element or with the background become new sources which may further interact with the canopy. The downward directed flux is combined with the appropriate hemispherical band of diffuse sky radiation. Upward directed flux is treated in a similar manner as diffuse sky radiation except the direction associated with each sector is the opposite from incoming radiation from that sector.

Geometrical measurements of a canopy provide a frequency distribution of foliage inclination angles. These can be integrated using Simpson's rule to obtain a cumulative frequency distribution that is normalized and partitioned into areas of equal probability. The interaction probabilities within each canopy layer are then calculated for both incident and emergent fluxes at each specified source direction. The following expression due to Idso and DeWit (1970) is used in the model:

$$P_0 = \left[1 - \frac{S \times OPM}{\sin \theta} \right]^{LAI/S} \quad (4.1)$$

$$PHIT = 1 - P_0, \quad (4.2)$$

where P_0 is the probability of a gap, P_{HIT} is the probability of an interaction, OPM is the mean canopy projection in the direction of the source, θ is the source inclination angle, and S takes a value from 0 to 1 depending on the density of the canopy. It is usually adjusted by optimizing the reflection prediction for either one view angle or a set of wave length since it does not change with either wave length or view angle. For large LAI, PHIT is insensitive to S and $S = 0.1$ is satisfactory.

A random number generator is used to generate a random number. A given source finds a gap in the top layer of the canopy if a random number is smaller than PHIT. The flux in this direction passes through the top layer and reaches the next canopy layer. The absence of a gap necessitates the determination of material type with which contact has been made. This is accomplished by sampling from the distribution of canopy constituents. The orientation of the leaf is determined by sampling from the inclination distribution and a uniform azimuthal distribution. These parameters determine the direction cosines of the leaf from which the angle between the leaf and the source is determined. The optical properties of the leaf are then utilized to calculate the flux exiting the leaf in all directions. Each sector of the hemisphere on the reflecting side of the leaf receives flux for each wavelength according to the equation,

$$I = \frac{I_o}{18} \rho \sin(\theta_{LS}) (\sin^2 \theta_2 - \sin^2 \theta_1) \quad (4.3)$$

I_o = source spectral flux

ρ = material spectral reflectance

θ_{LS} = angle between the leaf and the source

θ_2, θ_1 are the inclination angles defining the hemispherical band

The solid angle sectors receiving reflected and transmitted flux are defined in the same manner as for canopy flux sources only extended to include the entire sphere about the leaf. The leaf is not necessarily horizontal so a

sector receiving reflected flux from the leaf is not necessarily directed upward with respect to the local vertical. Hence, the direction cosines of the flux sector are rotated to the local vertical system and the flux pooled with the flux in the appropriate source band. Transmitted flux is calculated and treated in the same manner as reflected flux.

Flux which passes through a gap or is reflected or transmitted downward from an upper layer of the canopy interacts with the next lower layer.

Flux which reaches the soil surface is reflected into each of the upward directed source bands according to the equation:

$$I = I_o \rho_s \sin(\theta) (\sin^2 \theta_2 - \sin^2 \theta_1) \quad (4.4)$$

where

ρ_s = soil spectral reflectance

θ = source inclination angle

Upward directed flux from a lower layer of the canopy or from the background reaches the next higher layer and may interact with it. The upward directed flux from the top layer escapes the canopy.

The interaction procedure continues until the level of flux in any source direction within any layer is below a threshold value. The flux exiting the canopy into each of the bands is separately accumulated. The ratio of the flux intercepted by a sensor placed within one of these bands to the global radiation intercepted by a vertical sensor with the same field of view gives the canopy directional reflectance.

Several iterations through the model will reduce the standard error of the predicted reflectance. For each pass, it may be desired to generate random vectors, e.g., constituent optical properties or global radiation flux, to drive the model. The model is capable of generating random vectors from a

multivariated normal distribution or will use a mean vector on each pass if desired.

Detailed calculations were made for the case when the canopy is modeled as a three-layered vegetation ensemble, with each layer containing two types of vegetation elements which are assumed to be Lambertian scatterers. The model was evaluated (Smith and Oliver, 1974) against measured data for shortgrass prairie vegetation with a leaf area index of 6.5. For a vertical view angle, agreement is good except for the chlorophyll absorption band where the model predicts lower reflectance. Smith and Oliver suggest that this discrepancy may be due to their assumption of the surfaces being Lambertian whereas there is some indication that the reflection characteristics are non-Lambertian in regions of high absorption. Off-angle predictions of the model are qualitatively correct, but do not display the same precision as the vertical view case. The comparison also suggests that the Smith and Oliver model compares more favorably with the experimental result than those based on the Kubelka-Munk theory.

The model has also been used to understand the observed wide variability in diurnal reflectance (solar zenith angle dependence) for lodgepole pine and two grass canopies (Kimes, Smith, and Ranson, 1980). The model correctly simulated this variability and suggests that this variation is caused by variations in anisotropic sky irradiance, canopy component geometry and optical properties, and the type of reflectance measurement.

A major difficulty of the current implementation is the pooling of the outgoing radiance into θ directions only. That is, outgoing azimuthal directions are averaged. It should be noted, however, that incoming source azimuth directions are included. The model requires considerable time for the Monte Carlo analysis of a canopy. In principle, however, the approach is an accurate representation of the radiative transfer processes occurring within a plane-

parallel medium. A deterministic version of the model that included outgoing azimuthal dependence could greatly enhance the utility of this approach.

(2) Ross and Nilson Model

This model is again for plane parallel canopy and has been developed by two Estonian investigators Ross and Nilson (1966; see also Ross, 1975 for other references). They consider plant canopy as a turbid anisotropic and statistically homogenous (in the horizontal direction) layer and the radiation field inside the canopy is determined by numerically solving a modified radiative transfer equation.

They describe the geometrical structure of the canopy as a collection of numerous thin horizontal laminae. The thickness of each lamina is selected so that a ray propagating in the vertical or nearly vertical direction intercepts only one vegetation element; in this way the number of scattering events in each laminae is at most 1.

They further assume that each lamina has sufficiently large horizontal surface such that it contains a large number of vegetation elements and one can assume a statistical distribution of the spatial orientation of the leaves over that surface. This distribution is assumed to be the same for all laminae.

The vegetation canopy geometry is characterized by:

(a) $u_k(z)$: the amount of vegetation elements of type k in unit layer volume at distance z in the canopy. The leaf area index L_k for vegetation element of type k is related to u_k by

$$L_k = \int_0^h u_k(z) dz \quad (4.4)$$

where h is the height of the canopy.

(b) $g_k(z, \vec{r})$: the distribution density of the normal to the top surface of the vegetation elements of type k in a direction \vec{r} . This is used to calculate the so called G function which may be interpreted as the mean projection of a unit foliage area in a particular direction. They derive a radiative transfer equation not for a volume element but for each elementary lamina using intensities averaged over the coordinates x and y (z is the direction perpendicular to the canopy). This is the equation which is solved to calculate canopy reflectance.

In their formalism, the optical properties of the vegetation elements are defined by general wavelength dependent scattering functions. However, because of the difficulty in solving the general problem, in detailed calculations, they assume identical scattering properties for the two leaf surfaces and use reflection and transmission coefficients as the optical property characterizing the vegetation elements. They also assume that g_k is independent of z , i.e. distribution function of the leaf orientations is almost independent of z .

They investigated in detail three cases of leaf orientations: (1) all leaves horizontal, (2) all leaves vertical and (3) leaf normals uniformly distributed. For case (1) the radiative transfer equation can be analytically solved with reflectance exponentially decaying with LAI. For other cases, the equation has to be solved numerically.

On the basis of limited available published work in the english language, it appears that the model seems to agree well with the experimental results. However, because of the assumptions on basic laminae, the model can not take into account row effects without some drastic modifications.

(3) Adding Method

Very recently Cooper, Smith, and Pitts (1982) proposed a model based on a so called adding method (Van de Hulst, 1980), in which they do not directly

solve the radiative transfer equation. Instead, they divide the canopy into a set of appropriate sublayers, with each sublayer's reflectance properties characterized by two matrix operators R and R' (one for the top and the other for the bottom of the sublayer), and transmittance properties by operators T and T' . The elements of these operators give the flux transferred from one direction to another. Elements of R for the top sublayer will by definition give the canopy reflectance.

Explicit expressions for these optical operators are calculated numerically in terms of the optical properties of the canopy components and geometrical properties of the canopy is assumed to comprise of individual Lambertian components characterized by reflection, transmission, and absorption coefficients. The components are assumed to be oriented isotropically with respect to the azimuth but a leaf slope distribution $f(\theta_1)$ is specified. The other parameters used are LAI and reflectance of soil.

The leaf angle distribution and LAI together with the Lambertian assumption, are used to calculate the proportion of the incident flux which does not interact with the canopy layer, probability of gap (see Eq. (4.1)) and to compute the total canopy projected area in the directions of incident and exitant flux. These calculations coupled with optical properties of the vegetation elements are used to calculate optical matrix operators.

It should be noted that the size of various matrix operator and computer storage requirement will increase rapidly as the number of discrete ranges in which the incident and view directions are discretized is increased. (discretizing interval is decreased). In the initial implementation, the sun zenith and view angles ranged from 5 to 85 degrees and were discretized

in 10 degrees increment leading to 9×9 matrix operators.

The model has been checked against the reflectance measurement for a Blue grama and soybean canopies and against the Suits model. It appears that the model generally gives good results at infrared wavelengths and overestimates the reflectance at visible wavelength for soybean, particularly as compared to the Suits model. For Blue grama it yields excellent results. The authors suspect that the discrepancy is because the model is sensitive to asymmetrical scattering properties of the leaves; in the simulations, ρ and τ for the leaves were assumed independent of leaf side which is probably true for Blue grama but not for soybean. Also, for soybean the assumed form of probability of gap may be less valid than that for Blue grama.

We conclude the discussion of this model by pointing out that the model does not take into account the row effects and at this time it is not clear how it could be incorporated in the model.

(4) Weinman and Guetter Model

In this model proposed at the same time as the Suits model (Weinman and Guetter, 1972) the canopy is also idealized as a plane parallel medium. The radiative transfer equation is solved using the method of discrete coordinates (Chandrasekhar, 1950; Lenoble, 1956) and four-point quadrature. The phase function is represented essentially as a series of Legendre polynomials (series is truncated to include only the first four terms).

In their model they emphasized the need for the reduction of the number of parameters to characterize the optical properties of the vegetation canopy and the desirability of including atmosphere in the same model while describing the penetration of solar irradiance. Their model requires only three parameters to describe either the canopy (modeled as a single layer) or the atmosphere.

These parameters are:

a: albedo for single scattering (ratio of scattering to scattering and absorption).

τ : optical depth or LAI

g: asymmetry factor which is a measure of the deviation of scattering from an isotropic scattering.

They found that the data on corn canopy reflectance at 1μ wavelength used by Allen, Gayle, and Richardson (1970) can be fitted very well with their model with the following parameters:

Layer	height (m)	τ	a	g
atmosphere	$2.5 < z < 3000$	$0.087 > \tau > 0.0$	1.0	0.75
corn canopy	$0.0 \leq z \leq 2.5$	$3.17 > \tau > .08$	0.98	-0.44

(Note that the value of $a = 1$ implies no absorption, which is consistent with the conclusion of Allen, Gayle and Richardson (See Section III).

They emphasize that the three parameters can be derived from laboratory measurements of the optical properties of the leaves and canopy geometry.

They also give the following relations between these parameters and those occurring in Duntley equations (eq. (2.1.3).

$$2(1-a) = \alpha / (k + S_1 + S_2) \quad (4.5a)$$

$$\frac{3}{4} (1 - ag) - (1 - a) = \gamma / (k + S_1 + S_2) \quad (4.5b)$$

$$d\tau = (k + S_1 + S_2) d(\text{LAI}) \quad (4.5c)$$

This reviewer is not aware of any other comparison of the prediction of the model with other measurements. Therefore, its potentials and limitations can not be assessed. It appears that its extension to include the row effect

may be difficult, and may have to be done by sacrificing its simplicity.

(5) Welles and Norman Model

All the models discussed so far idealized the vegetation canopy as a parallel plane medium consisting of layers. We will now very briefly discuss a model which is truly three dimensional. It was proposed in 1979 (Welles and Norman, 1979) and seems to be very promising. Unfortunately details of the model have not yet been published. Therefore, one is unable to make a definitive assessment of the potentials and limitations of the model.

In this model, a canopy is modeled to consist of a finite number of a three dimensional geometrical figure like an ellipsoid or a cylinder which could be spaced in one of many patterns - regularly spaced, densely grouped (including overlapping) at regular spacing, sparse groups widely separated - to more realistically reflect a particular vegetation including those planted in rows. Within each of these geometrical figures, the foliage could be chosen to be distributed randomly or in a non-random fashion (e.g. different foliage density in the interior of the figure than on its periphery). This density distribution is chosen to possess the given foliage angle distribution. Once the foliage has been distributed in each geometrical figure, the attenuation of incident beam as it traverses the collection of figures is calculated. This in turn is used to numerically calculate the diffuse radiation by using a scattering phase function, reflecting the optical properties of the canopy elements. In visible region, the assumption of single scattering is used while in infrared region multiple scattering is allowed.

The next step which is rather innovative is to "transform" each point in the finite array of geometrical figures to an equivalent infinite plane parallel

medium canopy by choosing a depth in the plane parallel canopy that has the same diffuse penetration probability considering both the upward and downward radiation flux. That is, for an arbitrary point F in the array, a point H, in an infinite one layer canopy, is chosen, defined by its depth in the layer, such that the transmission at F in the upper (lower) hemisphere is the same as upward (downward) transmission of flux at point H. Similar equivalence is done for reflectance. Once equivalence has been established for all points in the array, the plane parallel medium is used to calculate bidirectional reflectance. Sunlit and shaded leaves are treated separately.

(6) Other Multi-dimensional Models

As plants grow, they possess characteristic shapes that govern the spatial arrangement of their reflective matter. Further, these shapes obscure varying amounts of soil or understory vegetation and also cast shadows on the soil, understory, or other plants. There are a set of other multi-dimensional models (as contrasted to plane parallel canopy case) in which geometric form factors for the rows and an analysis of shadowing play a dominant part. These models employ primarily geometric optics and multiple scattering is ignored. Egbert (1976, 1977) developed a model which allows calculation of optical bidirectional reflectance from shadowing parameters of surface projections or perturbations. Strahler and Li (1981, Li, 1981) model low density timber stands as a collection of randomly spaced cones. Each cone has a fixed base/height ratio, and is taken to be a flat Lambertian reflector which absorbs visible wave length differentially. Tree heights are assumed to be log normally distributed and tree counts from pixel to pixel vary according to Poisson distribution. This model can be inverted to yield tree heights and spacing from a remotely sensed reflectance data.

Other geometrical models have been proposed by Jackson et al (1979b) and Richardson et al (1975).

These models have their role in the crop canopy reflectance modeling but perhaps not as much as the ones which include multiple scattering.

V. Summary and Strategic Recommendations

In the last three sections we have provided an overview of the radiative transfer theory and how it has been used to calculate the canopy reflectance given the properties of the canopy constituents and canopy geometry. Each approach to its use has led to a model for canopy reflectance. All the evidence collected, to support various models, suggest that radiative transfer theory, which is a macroscopic approximation to the interaction of radiant energy with matter, is applicable to crop canopy reflectance. Thus, it should continue to play its pivotal role in crop canopy reflectance modeling.

Most of the models, with the exception of those based on the KM theory and Suits model, have been tested mostly by their authors using limited experimental data base. They do generally agree with the observations used in their testing (otherwise it is reasonable to conjecture that the model would not have seen the publisher's ink). To determine their range of applicability and relative merits and limitations, it is desirable that a uniform and extensive data base for 3 or 4 crops, in different stages of development and planted in different geometric configurations, be made available for exhaustive testing of various models either by the authors or by a centralized testing group. The data base should include measured values of the agronomic, optical, and geometric variables characterizing the canopy as well as measured reflectance for a set of incident radiation angles, view angles, and atmospheric conditions. This testing if implemented will clearly establish which model could or should not be used in a given situation.

Even without this test, it appears that none of the models will work for all crops and for all conditions. Therefore, parallel to the testing, development of crop reflectance models should continue. Here, though, in principle, it is desirable to develop as many models as possible, in practice one may have to choose only a few. If so, it appears that the best strategy

will be to develop two types of models.

One model should be simple and analytical (or at least semi-analytical) which is easily comprehensible and could be used with limited computing power and time to calculate reflectance from known canopy properties for a fairly large set of crops and canopy geometries. A model adapted from Suits model seems to be a good candidate. This modified model should allow inclusion of unequal optical properties of vegetation elements on its two sides, various leaf angle distribution functions, and row effects. As discussed in Section III, these inclusions have already been done separately, but not in one encompassing and tested model.

The other model should be a comprehensive one and be capable of incorporating detailed properties of the canopy, without making any significant approximations in the process. This model should be characterized not by its simplicity and comprehensibility but by its accuracy in calculating the crop reflectance from canopy parameters. Such a model will, of necessity, require numerical solution of the radiative transfer equation and hence may not be very kind to computer storage and time requirements. The more difficult part of the development effort of this model is the characterization of scattering phase function in terms of canopy variables and not the procedure for iterative numerical solution of the radiative transfer equation. The latter technology has been fairly well developed in many other applications of the radiative transfer equation.

Since the simpler model is an adaption of several existing models, it could become available in a comparatively short term (1-2 years). The comprehensive model involves some original development and may require a longer term effort.

All the canopy reflectance models proposed to date have a major deficiency. They do not include time as an implicit variable. That is, if one wants to calculate crop reflectance at different times (stages of development), one has

to input in the model canopy variables for each time. Getting information for this input is rather time consuming. Therefore, it is desirable that a reasonable effort be made to combine a canopy reflectance model (preferably a simple one like the Suits model) with a vegetation growth model. Such a model will then provide a natural link to the temporal profile models (Badhwar, 1980; Badhwar, Austin and Carnes, 1982; Badhwar and Henderson, 1981) which have been used for automatic crop classification and crop emergence date and growth stage determinations.

To date most of the models have been used as a research tool and for understanding, i.e. for defining the proper instrumentations (e.g. spectral bands of sensors), in interpreting data, for assisting in the identification of appropriate transform of reflectance in various wavelengths which may be insensitive to some canopy parameters, and for identifying potential causes of abnormal observations.

They have also been the potentials for 'forecasting' reflectance for hitherto not tried set of canopy parameters. However, in light of the overall goal of crop identification and crop growth stage and quality determination from the reflectance data, it is imperative that various crop reflectance models be investigated to assess their capabilities in correctly and uniquely determining the canopy parameters of importance like LAI, solar radiation intercepted (SRI), etc. from the reflectance data. In other words, they should be tested for their invertability. As part of this testing one should also determine how sensitive the results are to the variations in the reflectance data. Also, the invertability of reflectance for a given λ vs. that for a linear combination of reflectance for many wavelength bands (e.g. Kauth-Thomas Greenness) should be investigated. Like with any such endeavor, first simpler models should be tested for inversion, including those which can

be semi-analytically inverted, followed by complex models requiring numerical inversion. Mathematical techniques for the inversion or what is more commonly referred to as systems identification are generally available but what is required is their adaption in the present context.

In connection with the inversion, we should point out that it is possible that the present models, especially the more complex ones, may not be attractive from the inversion point of view. It is therefore, desirable to look into some reflectance models which are invertible but have not yet been tried in the context of crop canopies. One such model is the so called multi-thin layer model which has been used in designing optical system which modifies spectral composition of an incident radiation in a given manner (see e.g. Dobrowolski, 1981).

As the name implies, in this model, the optical system is considered as a set of thin film layers system. The main physical properties that can be modified by such a system are transmittance, reflectance, absorption, and polarization. The system is designed to meet a required performance for one or more of these optical properties at a selected wavelength or in a certain wave length region. That is, the method can be used to determine the construction parameters of a thin film device, namely, refractive index n , absorption coefficient k , and thickness d . The method has been demonstrated to be a practical one for most coating systems.

To adapt the technique for vegetation, for a given reflectance $R(\lambda, \theta, \phi)$ which depends on wave length λ , and view direction (θ, ϕ) , one will use the method to calculate the parameters $n(\lambda)$, $k(\lambda)$ and $d(\lambda)$. One hopes that the values of these parameters will be sufficiently different for different crops to allow crop identification. It is conceivable that one may need to use R at different times and/or a multilayer system with a set of parameters n , k , and d which vary continuously as a function of distance perpendicular to the thin layer.

At the outset it should be pointed out that there is an important difference between the thin film devices for which the method has been developed and the canopy system. This may lead one to question its validity. A canopy structure causes an incident radiation to scatter in all possible directions instead of being reflected in the specular direction as for a thin film system. Ignoring this difference one may go ahead and obtain an equivalent multilayer system which can generate for a given incident radiation the specified amount of scattered radiation in a given direction. For a different view angle, another set of parameters for the equivalent system may be obtained. If the two sets of parameters are close enough, then there is a high probability that as long as two crops have two different spectral characteristics, two different equivalent systems may be generated to tell them apart (a successful crop identification technique). If not, one may have to modify the thin film technique to include scattering.

We conclude this section by enumerating a set of modeling areas which are relevant to the crop canopy reflectance, but were outside the scope of this review. They should be looked into more detail to determine a desirable strategy for further work. These areas are as follows:

- (1) Modeling of reflectance from single vegetation component, e.g. leaf as a function of wave length as well stress condition. This has the potential of allowing stress condition identification from the reflectance data.
- (2) Modeling of reflectance in the thermal infra-red region (3 -20 μm). This is rather useful in light of measurements made in this band by Thematic Mapper on board Landsat-D. Also, it could lead to the assessment of water status of the bare soil as well as vegetation canopy from the reflectance data which, if known during the early stages of growth, could be useful in predicting the maximum potential yield.

(3) Modeling of reflectance (back scattering) in the microwave region.

This could be useful in the identification of small grains crops and the determination of surface soil moisture, plant moisture and the leaf area index.

(4) Modeling of atmosphere separately as well as in tandem with the crop canopy reflectance. This obviously is very relevant since the eventual goal of the modeling effort is to use satellite based reflectance data to assess vegetation.

REFERENCES

1. Allen, J.H. Jr., and Brown, K.W. (1965) 'Short Wave Radiation in a Corn Crop'. Agron. J. 57, 575-580.
2. Allen, W.A., Gayle, T.V., and Richardson, A.J. (1970) 'Plant-Canopy Irradiance Specified by the Duntley Equations'. J. Opt. Soc. Am. 60, 372-376.
3. Allen, W.A., and Richardson, A.J. (1968) 'Interaction of Light with a Plant Canopy'. J. Opt. Soc. Am. 58, 1023-1028.
4. Badhwar, G.D. (1980) 'Crop Emergence Date Determination from Spectral Data'. Photogramm. Eng. Remote Sens. 46, 369-377.
5. Badhwar, G.D., Austin, W.W. and Carnes, J.G. (1982) 'A Semi-Automatic Technique for Multitemporal Classification of a Given Crop Within a Landsat Scene'. Pattern Recognition, 15, 217-230.
6. Badhwar, G.D., and Henderson, K.E. (1981) 'Estimating Development Stages of Corn from Spectral Data - An Initial Model'. Agron. J. 73, 748-755.
7. Bunnik, N.J.J. (1978) 'The Multispectral Reflectance of Shortwave Radiation by Agricultural Crops in Relation with Their Morphological and Optical Properties'. H. Vennman & Zonen, B.V. Wageningen. 175 p.
8. Bunnik, N.J.J., and Verhoef, W. (1974) 'Measurements on a Wheat and a Grass Canopy for Some Stages of Growth'. NIWARS 23, Delft, The Netherlands.
9. Chance, J.E., and Cantu, J.M. (1975) 'A Study of Plant Canopy Reflectance Models'. Unpublished Report, Pan American University, 61p.
10. Chance, J.E. (1977) 'Applications of Suits Spectral Model to Wheat'. Remote Sens. Environ. 6, 147-150.
11. Chance, J.E., and LeMaster, E.W. (1977) 'Suits Reflectance Models for Wheat and Cotton: Theoretical and Experimental Tests', Appl. Opt. 16, 407-412.
12. Chance, J.E., and LeMaster, E.W. (1978) 'Plant Canopy Light Absorption Model with Application to Wheat'. Appl. Opt. 17, 2629-2636.
13. Chandrasekhar, S. (1960) 'Radiative Transfer', Oxford, Clarendon Press, 393 p.
14. Colwell, J.E. (1974) 'Vegetation Canopy Reflectance', Remote Sens. Environ. 3, 175-183.
15. Cooper, K., Smith, J.A., and Pitts, D. (1982) 'Reflectance of a Vegetation Canopy Using the Adding Method'. Preprint.
16. Curran, P. (1981) 'The Relationship Between Polarized Visible Light and Vegetation Amount'. Remote Sens. Environ. 11, 87-92.

17. Dobrowolski, J.A. (1981) 'Versatile Computer Program for Absorbing Optical Thin Film Systems'. Appl. Opt. 20 , 74-81.
18. Duntley, S.Q., (1972) 'The Optical Properties of Diffusing Materials'. J. Opt. Soc. Am. 32 , 61-70.
19. Egan, W.G., (1978) 'Optical Depolarization Properties of Surfaces Illumination by Coherent Light'. Appl. Opt. 7 , 1529-1534.
20. Egan, W.G. (1970) 'Optical Stokes Parameters for Farm Crop Identification' Remote Sens. Environ. 1 , 165-180.
21. Egbert, D.D. (1976) 'Determination of the Optical Bidirectional Reflectance From Shadowing Parameters'. Ph.D. Thesis, Univ. of Kansas.
22. Egbert, D.D. (1977) 'A Practical Method for Correcting Bidirectional Reflectance Variations'. Proc. Symp. Mach. Process. of Remote Sens. Data; 178.
23. Ellenson, J.L., and Amundson, R.G. (1981) 'Delayed Light Imaging for the Early Detection of Plant Stress'. Science 215 , 1104-1106.
24. Hatfield, J.L. (1981) 'Spectral Behavior of Wheat Yield Variety Traits', Photogramm. Eng. Remote Sens. 47 , 1487-1491.
25. Van de Hulst, H. (1980) 'Multiple Light Scattering', London, Academic Press.
26. Idso, S.B., and DeWit, C. T. (1970) 'Light Relations in Plant Canopies. Appl. Opt. 9 , 177-184.
27. Ishimaru, A. (1978a) 'Wave Propagation and Scattering in Random Media Vol. 1. Single Scattering and Transport Theory', Academic Press, New York.
28. Ishimaru, A. (1978b) 'Wave Propagation and Scattering in Random Media. Vol. 2. Multiple Scattering, Turbulence, Rough Surfaces and Remote Sensing'. Academic Press, New York.
29. Jackson, R.D., Pinter, P.J. Jr., Idso, S.B., and Reginato, R.J. (1979a) 'Wheat Spectral Reflectance: Interactions Between Crop Configuration, Sun, Elevation, and Azimuth Angle'. Appl. Opt. 18 , 3730-3732.
30. Jackson, R.D., R.J. Reginato, Pinter, R.J., and Idso, S.B., (1979b) 'Plant Canopy Information Extraction from Composite Scene Reflectance of Row Crops'. Appl. Opt. 18 , 3775-3782.
31. Kimes, D.S., and Smith, J.A., (1980) 'Simulation of Solar Radiation Absorption in Vegetated Canopies'. Appl. Opt. 19 , 2801-2811.
32. Kimes, D.S., Smith, J.A., and Ranson, K.J., (1979) 'Terrain Feature Canopy Modeling' Final Report, U.S. Army Research Office, Grant No. DAAG 29-78-G0045, 160 pp.
23. Kimes, D.S., Smith, J.A., and Ranson, K.J. (1980) 'Vegetation Reflectance Measurements as a Function of Solar Zenith Angle'. Photogramm. Eng. Remote Sens., 46 , 1563-1573.

24. Kortüm, G. (1969) 'Reflectance Spectroscopy', New York Springer-Verlag.
25. Kubelka, P., and Munk, F., (1931) 'Ein Beitrag zur Optik der Farbanstriche', Z. Tech. Physik, 12, 593.
26. LeMaster, E.W., and Chance, J.E., (1978) 'Further Tests of the Suits Reflectance Model', Proc. 11th Int. Symp. on Remote Sensing & Environ. Univ. of Michigan, Ann Arbor, 1, 703-716.
27. LeMaster, E.W., Chance, J.E., and Wiegand, C.L. (1980), 'A Seasonal Verification of the Suits Spectral Reflectance Model for Wheat'. Photogramm. Eng. and Remote Sens. 46, 107-114.
28. Lenoble, J. (1956) 'Application de la methode de Chandrasekhar a l'etude du rayonnement diffuse dans le brouillard et dans la mer'. Rev. Opt. 35, 1-17.
29. Li, X. (1981) 'An Inversible Coniferous Forest Canopy Reflectance Model'. M.A. Thesis, Univ. of Calif. at Santa Barbara, 108p.
30. Park, J.K., and Deering, D.W. (1982) 'Simple Radiative Transfer Model for Relationships between Canopy Biomass and Reflectance'. Appl. Opt. 21, 303-309.
31. Ranson, K.J., Vanderbilt, V.C., Biehl, L.L., Robinson, B.F., and Bauer, M.E., (1982) 'Soybean Canopy Reflectance as a Function of View and Illumination Geometry'. LARS Report No. SR-P2-04278, April, 1982.
32. Richardson, A.J., Wiegand, C.L., Gausman, H.W., Cuellar, J.A., and Gerbermann, A.H., (1975) 'Plant, Soil, and Shadow Reflectance Components of Row Crops'. Photogramm. Eng. and Remote Sensing, 41, 1401-1407.
33. Ross, J. (1976) 'Radiative transfer in Plant Communities in Vegetation and the Atmosphere'. Vol. 1 Principles (ed. J.L. Monteith), Acad. Press London, 13-55.
34. Ross, J.K., and Nilson, T.A., (1966) 'A Mathematical Model of the Radiation Regime of Vegetation' in Actinometry and Atmospheric Optics' V.K. Pyldmaa (Ed). Transl. by Israel Progr. Sci. Transl. Jerusalem, 1971.
35. Sačowski, F.G., and Malila, W.A., (1977) 'Investigation of Techniques for Inventorying Forested Regions, Vol. I. Reflectance Model and Empirical Multispectral Analysis of Forest Canopy Components'. Final Report, NASA Contract NAS9 14988, ERIM, Ann Arbor, MI.
36. Slater, P.N. (1980) 'Remote Sensing - Optics and Optical Systems', Addison-Wesley, Reading, Mass.
37. Smith, J.A., and Oliver, J.E., (1972) 'Plant Canopy Models for Simulating Composite Scene Spectroradiance in the 0.4 to 1.05 micrometer Region'. Eighth Symp. on Rem. Sens. of Environ. Univ. of Michigan, Ann Arbor, 2, 1333-1353.
38. Smith, J.A., and Oliver, R.E., (1974) 'Effects of Changing Canopy Directional Reflectance on Feature Selection'. Appl. Opt. 13, 1599-1604.

39. Smith, J.A., and Ranson, K.J., (1979) 'MRS Literature Survey of Bidirectional Reflectance and Atmospheric Corrections II. Bidirectional Reflectance Studies Literature Review'. NASA-Goddard Report, August, 1979.
40. Strahler, A.H., (1981) 'Stratification of Natural Vegetation for Forest and Rangeland Inventory Using Landsat Digital Imagery and Collateral Data'. Int. J. Remote Sens., 2, 15-41.
41. Strahler, A.H., and Li, X., (1981) 'An Invertible Coniferous Forest Canopy Reflectance Model'. Proc. 15th Int. Symp. Remote Sens. Environ. Ann Arbor, MI.
42. Suits, G.H. (1972a) 'The Calculation of the Directional Reflectance of Vegetative Canopy'. Remote Sens. Environ., 2, 117-125.
43. Suits, G.H., (1972b). 'Azimuthal Variations in Directional Reflectance of Vegetative Canopies'. Remote Sens. Environ. 2, 175-182.
44. Suits, G.H., (1972c) 'Prediction of Directional Reflectance of a Corn Field Under Stress'. 4th Ann. ER. Program Review, Jan. 17-21, 11p.
45. Suits, G.H., and Safir, G. (1972) 'Verification of a Reflectance Model for Mature Corn with Applications to Corn Blight Detection'. Remote Sens. Environ. 2, 183-192.
46. Suits, G.W. (1981) 'The Extension of a Uniform Canopy Reflectance Model to Include Row Effects'. ERIM Rept. No. SR-EI-04065, 49p.
47. Tucker, C.J., (1978) 'Post Senescent Grass Canopy Remote Sensing'. Remote Sens. Environ., 7, 203-210.
48. Vanderbilt, V.C., (1980) 'A Model of Plant Canopy Polarization Response'. Proc. Symp. on Machine Processing of Remotely Sensed Data, pp. 98-108.
49. Vanderbilt, V.C., Biehl, L.L., Robinson, B.F., Bauer, M.E., and Vanderbilt, A.S., (1980) 'Linear Polarization of Light by Two Wheat Canopies Measured at Many View Angles'. LARS Technical Report 090981, September, 1980.
50. Vanderbilt, V.C., Kollenkark, J.C., Biehl, L.L., Robinson, B.F., Bauer, M.E., and Ranson, K.J., (1981) 'Diurnal Changes in Reflectance Factor Due to Sun-Row Direction Interactions'. LARS Rept. No. SR-P1-04140, September, 1981.
51. Verhoef, W. and Bunnik, N.J.J., (1975) 'A Model Study on the Relations Between Crop Characteristics and Canopy Reflectance. 1. Deduction of Crop Parameters from Crop Spectroreflectance Data 2. Detectability of Variations in Crop Parameters by Multispectral Scanning'. NIWARS Publication No. 13, 89 p.
52. Verhoef, W., and Bunnik, N.J.J., (1976) 'The Spectral Directional Reflectance of Row Crops. 1. Consequence of Non-Lambertian Behavior for Automatic Classification 2. Measurements on Wheat and Simulations by Means of a Reflectance Model for Row Crops'. NIWARS Publication No. 35, 135 p.
53. Verhoef, W. and Bunnik, N.J.J., (1981) 'Influence of Crop Geometry on Multispectral Reflectance Determined by the Use of Canopy Reflectance Models'. Proc. Inter. Colloq. on Signatures of Remotely Sensed Objects. Avignon, France, September 8-11, 1981. pp. 273-290.

54. Weinman, J.A., and Guetter, P.J., (1972) 'Penetration of Solar Irradiances through the Atmosphere and Plant Canopies'. J. Appl. Meteor., 11 , 136-140.
55. Welles, J.M., and Norman, J.M., (1979) 'General Radiative Transfer Model for Random and Non-random Canopies'. Fourteenth Conf. on Agriculture and Forest Meteorology and Fourth Conference on Bio-Meteorology 205-206.